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Simple Markovian Queueing Models Using Solving of Queue Problem

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ABSTRACT

The theory of queuing is a calculable way for modelling many more types of queuing systems mathematically. These models can be used to forecast how the system will respond to changes in demand. The study of queues and queuing behavior is known as queuing theory. We introduced single-single queuing models (SSQM) that use interval areas to deal with indefinite parameters in this research. The arrival and facilities charges are well-thought-out interval areas, and we used a new interval arithmetic approach to investigate the features of queuing models. A numerical demonstration is also used to validate the suggested model.

Keywords: Single-server queuing model (SSQM), Queuing theory (QTs), Mathematical modeling, Arrival state and rate, Queuing model (QMs).

I. INTRODUCTION

The QMs is a mathematical model that has a large number of applications in service companies such as healthcare and manufacturing firms, in which different sorts of clients are served by different types of servers according to different queue disciplines. QTs is very useful in computer systems for estimating the value of particular computer performance measures [1]. The mathematical study of waiting lines, or queues, is known as queuing theory. A model is built in QTs so that queue lengths and waiting times may be predicted. QTs is usually regarded a branch of OR since the results are regularly used to make business decisions about the resources required to provide a service [2]. A K Erlang's research into models to represent the Copenhagen telephone exchange gave rise to queueing theory. Telecommunications, traffic engineering, computing, and factory, shop, office, and hospital design have all benefited from the concepts. To take care of indefinite parameters, we proposed a single-server QMs with interval areas in this study [3].

II. SYMBOL & NOTATIONS

n = Total number of clients in the system, including those who are waiting and those who are being served.
μ = Per unit of time, the average number of customers served.
λ =Average number of customers arriving per unit of time.
C = The number of concurrent service channels.
L_S = The average number of customers in the system, both in line for service and waiting to be served.
L_q = The average number of consumers in line at any given time.
W_S = Average time a consumer spends in the system, both waiting and receiving assistance.
W_q = The average length of time a consumer waits in line.
P_n = Probability that there are n customers in the line.

III. BASIC QUEUING FORMULAS

The following are the outcomes of Little's rule:

$$L = \lambda W$$
$$L_q = \lambda W_q$$

The first pertains to the system, whereas the second applies to the queue, which is a component of the system. In the queue, another relevant relationship is:

$$W = W_q + \frac{1}{\mu}$$

………(1)

The above is intuitive it says that the system's mean wait is the sum of the queue's mean wait and the service time.

IV. QUEUEING NOTATION

Queues are represented using the following notation: A/B/c/K denotes the distribution of the inter-arrival time, B that of the service time, c that of the number of servers, and K that of the queue's maximum capacity, where A denotes the distribution of the inter-arrival time, B that of the service time, c that of the number of servers, and K that of the queue's maximum capacity. If K isn't specified, we'll assume 1 [4]. The exponential distribution is frequently referred to as M, which stands for Markov. As a result, an M/M/1 queue consists of one server (and one channel) with exponentially distributed interarrival and service times. An M/G/1 queue is one with three servers in which the inter-arrival time is exponentially dispersed and the service time is generally distributed, that is, the service time can take any shape [5]. A G/G/1 queue is one that has only one server and has any given distribution for both service and inter-arrival time [6].

V. MATHEMATICAL MODELING

First, we'll look at single-server queues with c = 1. They can be found in a wide range of production and service systems. For the M/M/1 queue, we can prove that (Ross, 2014)

$$L_q = \frac{\rho^2}{1-\rho}$$

For the M/G/1 queue, we can prove that

$$L_q = \frac{\lambda^2\sigma_s^2 + \rho^2}{2(1-\rho)}$$

The Pollazcek-Khintichine formula (named after its creators and discovered in the 1930s; Ross (2014)) [7] is what this is termed. We don't have a precise outcome for the G/G/1 queue. In industry, the following approximation (developed in Marchal (1976)) is widely used [8]:

$$L \approx \frac{\rho^2(1+C_s^2)(C_a^2+\rho^2 C_s^2)}{2(1-\rho)(1+\rho^2 C_s^2)}$$

………...(2)

In the above, if the mean rate of arrival is $\lambda$and σ_a^2denotes the variance of the inter-arrivaltime, then [9]:

$$C_a^2 = \frac{\sigma_a^2}{\left(\frac{1}{\lambda}\right)^2}$$

Similarly, if μdenotes the service rate and σ_a^2 denotes the variance of the service time, then:

$$C_s^2 = \frac{\sigma_s^2}{\left(\frac{1}{\mu}\right)^2}$$

Another approximation from Kraemer and Langenbach-Belz (1976) is also quite powerful [10]:

$$L_q \approx \frac{\rho^2(C_a^2+C_s^2)}{2(1-\rho)} g$$

……..(3)

where,

$$g = \exp\left(-\frac{2(1-\rho)(1-C_a^2)^2}{3\rho(C_a^2+C_s^2)}\right)$$

when $C_a^2 \leq 1$…….(4)

$$g = \exp\left(\frac{(1-\rho)(1-C_a^2)}{C_a^2 + 4C_s^2}\right)$$

when $C_a^2 > 1$…...(5)

Example 1: Consider the single-server queue below: the inter-arrival time is exponentially dispersed with a mean of 10 minutes, and the service time is also exponentially distributed with a mean of 8 minutes, find the (i) mean wait in the queue, (ii) mean number in the queue, (iii) the mean wait in the system, (iv) mean number in the system and (v) proportion of time the server is idle.

Solution: We have $M/M/1$ system. We also have

$$\lambda = \frac{1}{10}$$

$$\mu = \frac{1}{8}$$

Hence,

$$\rho = \frac{8}{10}$$

Then, number in the Queue

$$L_q = \frac{\rho^2}{1-\rho} = \frac{0.8^2}{1-0.8} = 3.2$$

Wait in the queue

$$W_q = \frac{L_q}{\lambda} = \frac{3.2}{\frac{1}{10}} = 32\ mins.$$

Wait in the system

$$W = W_q + \frac{1}{\mu} = 32 + \frac{1}{\frac{1}{8}} = 32 + 8 = 40\ mins.$$

Number in the system

$$L = \lambda W = \frac{1}{10} \times 40 = 4$$

Proportion of time the server is idle

$$1 - \rho = 1 - \frac{8}{10} = 0.2$$

Example 2: Consider the single-server queue below: The service time has a uniform distribution with a maximum of 9 minutes and a minimum of 7 minutes, while the inter-arrival time is exponentially distributed with a mean of 10 minutes, find the (i) mean wait in the queue, (ii) mean number in the queue, (iii) the mean wait in the system, (iv) mean number in the system and (v) proportion of time the server is idle.

Solution: We have an $M/G/1$ system. We also have

$$\lambda = \frac{1}{10}$$

The mean service time will be

$$\mu = \frac{1}{8}$$

The variance of the service time,

$$\sigma_s^2 = \frac{(9-7)^2}{12} = \frac{1}{3}$$

Also,

$$\rho = \frac{8}{10}$$

Then, number in the queue

$$L_q = \frac{\lambda^2 \sigma_s^2 + \rho^2}{2(1-\rho)} = \frac{\left(\frac{1}{10}\right)^2 \times \frac{1}{3} + \left(\frac{8}{10}\right)^2}{2\left(1-\frac{8}{10}\right)} = 1.608$$

Wait in the queue

$$W_q = \frac{L_q}{\lambda} = \frac{1.608}{\frac{1}{10}} = 16.08\, mins.$$

Wait in the system

$$W = W_q + \frac{1}{\mu} = 16.08 + \frac{1}{8} = 24.08\, mins.$$

Number in the system

$$L = \lambda W = \frac{1}{10} \times 24.08 = 2.408$$

Proportion of time the server is idle

$$1 - \rho = 1 - \frac{8}{10} = 0.2$$

VI. CONCLUSION

A new strategy to solving the single and single-server queue is examined in this research. The novel interval arithmetic is used to obtain quality measures for single-server queues as interval numbers. The approach for solving the problem is demonstrated through numerical examples.

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