Basic Law of the Flat Interlocking of Involute Cylindrical Gears with Asymmetric Profiles

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ABSTRACT

This article is deduction of the necessary and sufficient condition for geometric synthesis of involute cylindrical gears with asymmetric profile of the teeth by the classic approach. The necessary condition is presented for slack-free engagement of the corresponding involutes along the initial circles and the crossing of the lines of engagement along the inter-center straight line, which is a new interpretation of the Willis's basic law of the flat engagement. Based on the theorem for reversing the direction of movement, two consequences are drawn, allowing generation of asymmetric tooth profile along various poloids and achieving qualitative indicators of engagement, which cannot be achieved with a symmetric profile.

Keywords: basic law of the flat interlocking, possibilities of generation, involute cylindrical gears, asymmetric profile

INTRODUCTION

The classic theory of gearing takes as inputs the parameters of the tool needed for production of the toothed wheels of the gear. For the geometric synthesis of an involute cylindrical gear, four output parameters are necessary: modulus of the gear - $m$, profile angle of the output contour - $\alpha$, coefficient of displacement of the output contour - $x$, the number of teeth of the wheels of the gear - $z$. These four parameters determine the axiomatic design of gear mechanisms under certain parameters of the tool. The use of asymmetric involute cylindrical gears eliminates the uncertainty of the selection of independent parameters and a "free geometric synthesis" can be implemented.

SUMMARY

Axiom of the tool grip for asymmetric profile of the teeth

The uniformity in the production of toothed wheels by the method of rolling is provided by introducing the so called "output contour" which is standardized [1,2,3,4].

The output contour is a rectilinear rack, whose sides are inclined at profile angle $\alpha=20^\circ$. The line along which the tooth thickness is equal to the width of the inter tooth space ($s=e=p/2$) is called pitch line.

The so called modified output contour also exists. The geometrical calculation of the involute gears is done on the basis of the standard elements of the contour. The output generating contour fills the inter tooth space of the output contour similar to shape and mold. The output generating contour serves for profiling the teeth of the tooth-cutting tools.

The tooth of the generating contour is divided into two by the pitch line - head and base. If the height of the head is equal to the height of the base, the generating contour is called equally high and if, at the pitch line the thickness of the tooth is equal to the thickness of the inter tooth space - evenly divided (uniform).

The generating contour is the orthogonal projection of the cutting edges of the tooth cutting tool - rack type on the front surface of the cut toothed wheel. It is the base of the tooth-cutting tool, however it does not account for the front and back angles, as well as other inherent characteristics of the tool.
In the engineering, an equally high contour is used - mill with open profile, while in precision engineering - mill with full profile is used.

Therefore, the cutting edges of the tool determine the shape and the nominal dimensions of the involute profiles of the tooth when wrapping the work-piece. This statement defines the tool for production of wheels using centroidal wrapping, to which one shift corresponds relative to the pitch circle. Therefore, the axiom of tool gearing for an asymmetrical profile is transformed in the form of:

**The generation of teeth with asymmetrical profile of involute cylindrical gears allows changing the module, the profile angles of the output contour and the coefficient of shifting in such proportions that the poloid circles of the gear wheels remain constant.**

For better clarity in the interpretation of the geometrical parameters of the gears, the following clarifying concepts are adopted - primary and secondary axis of symmetry of the output contour. "Primary axis of symmetry" is the axis dividing the tooth of the output contour into two parts with equal thicknesses. The asymmetrical tooth profile actually does not have such characteristics, these are adopted for the use of a certain analogy between the existing relationships in the literature, based on the established ones. In the correlations of the symmetric profile, the "+" sign refers to involute cylindrical wheels with external teeth, while the "-" sign refers to involute cylindrical wheels with internal teeth. Coefficient of displacement of the wheel with internal teeth means the displacement of the profile of the tool of the equivalent wheel with external teeth.

Apart from the axiom of instrumental gearing, another hypothesis set forth in the classical theory is the one of "congruence of the generating pairs". According to this hypothesis, congruent generating pairs are the pairs, whose generating surfaces can be brought into such a position that they coincided with each other, imposing one another at all points. From this hypothesis, it follows that the two wheels of the gearing are manufactured with one type of tool, while the asymmetric profiles of the toothed wheel are generated by the inter tooth spaces of the tool during the relative movement of the work-piece relative to the tool.

**Fig. 1. Output (OC) and generating contour (GC) of an asymmetric toothed profile in front section**

Fig. 1 shows an output contour of an asymmetric profile in the front section, which differs from the standard ones, by:

- The presence of two different profile angles \( \alpha, \alpha^* \), and equal pitch, both between the corresponding and between the opposite profiles. The toothed wheels generated this way are conventionally denoted as I;
the presence of equal pitch, but only between the corresponding tooth profiles. The gears generated this way are conventionally denoted as II. For this generation, the following necessary condition must be observed:

$$m_{t1}^* + m_{t2}^* = 2.m_t,$$

where $m_{t1,2}^*$ are the front modules of the output contour with non-uniform thickness, corresponding to profile angle $\alpha_t$, for wheels 1 and 2 of the gear (Fig.1).

$m_t$ is the front module of the output contour with non-uniform thickness, corresponding to profile angle $\alpha_t$ for wheels 1 and 2 of the gear (Fig.1).

By using the axiom for instrumental engagement and the hypothesis for congruency of the generating pairs, the following differences are found in the design of involute cylindrical gears with asymmetric profile and the determining of the thickness of the tooth along the pitch circle.

The determination of the thickness of the tooth of the toothed wheel on the pitch circle, when the contour is displaced, is:

- I possibility:

$$S_{t1,2} = \frac{\pi.m_t}{2} \pm x_{1,2}.m_t \left(\tan \alpha + \tan \alpha^* \right).$$

- II possibility:

$$S_{t1} = \frac{\pi.(m_t + m_{t1}^*)}{4} \pm x_1.m_t \left(\tan \alpha + \tan \alpha^* \right);$$

$$S_{t2} = \frac{\pi.(m_t + m_{t2}^*)}{4} \pm x_2.m_t \left(\tan \alpha + \tan \alpha^* \right); m_{t1}^* + m_{t2}^* = 2.m_t.$$

Theorem for reversing the direction of the movement in case of asymmetry of the tooth profile

After finding the basic geometric parameters of the tool, geometric synthesis of the involute gear with asymmetric profile of teeth follows. Let's hypothetically suppose that the gear with asymmetric teeth profile is generated and completely determined by two symmetrical gears. This is found easily, using the following relations for determination of the angles of engagement ($\alpha_{tw}$ and $\alpha_{tw}^*$):

$$\begin{array}{l}
\text{inv } \alpha_{tw} = \frac{2.(x_2 \pm x_1)}{z_2 \pm z_1} \tan \alpha + \text{inv } \alpha_t; \text{inv } \alpha_{tw}^* = \frac{2.(x_2 \pm x_1)}{z_2 \pm z_1} \tan \alpha^* + \text{inv } \alpha_t \\
\text{d}_{tw} = \frac{d_{tb1,2}^*}{\cos \alpha_{tw}}; \text{d}_{tw} = \frac{d_{tb1,2}^*}{\cos \alpha_{tw}},
\end{array}$$

where $\alpha_t$ and $\alpha_t^*$ are the profile angles of the tool, producing the wheels by the method of centroid wrapping of the front section;

$d_{tw1,2}$—diameters of the initial cylinders in the front section;

$d_{tw1,2}$—diameters of the main cylinders in the front section.
After defining the angles of engaging and the inter axis distances for both independent symmetrical speeds, these should also be determined for the asymmetric gear. If the correlations for their determination coincide, it follows that such gear consists of two symmetrical ones and is completely determined by them.

In order that the gear with asymmetric profile of the teeth is working, a slack-free engagement between the opposite profiles must be realized along the initial cylinders:

\[
(S_{tw1,2} + S^*_{tw1,2}) = \frac{\pi}{z_1}(d_{tw1} + d^*_{tw1}) = \frac{\pi}{z_2}(d_{tw2} + d^*_{tw2}),
\]

where \(S_{tw1,2}\) are the thicknesses of the asymmetric teeth in the front section, for wheels 1 and 2 of the gear, corresponding to profile angle \(\alpha_t\), along the initial cylinders;

\(S^*_{tw1,2}\) are the thicknesses of the asymmetric teeth in the front section, for wheels 1 and 2 of the gear, corresponding to profile angle \(\alpha^*_t\), along the initial cylinders.

The examining of the asymmetric tooth as composed of two symmetrical ones allows determining their thicknesses \((S_{tw1,2} \text{ and } S^*_{tw1,2})\) along the initial cylinders in the front section of the wheels:

- **I possibility:**

\[
S_{tw1,2} = \left\{ \frac{\pi}{2 \cdot z_{1,2}} \pm \frac{2 \cdot x_{1,2} \cdot \tan \alpha_t}{z_{1,2}} \pm \text{inv } \alpha_t \mp \text{inv } \alpha_{tw1,2} \right\} \cdot r_{tw1,2} ;
\]

\[
S^*_{tw1,2} = \left\{ \frac{\pi}{2 \cdot z_{1,2}} \pm \frac{2 \cdot x_{1,2} \cdot \tan \alpha^*_t}{z_{1,2}} \pm \text{inv } \alpha^*_t \mp \text{inv } \alpha^*_{tw1,2} \right\} \cdot r_{tw1,2} ;
\]

- **II possibility:**

\[
S_{tw1,2} = \left\{ \frac{\pi}{2 \cdot z_{1,2}} \pm \frac{2 \cdot x_{1,2} \cdot \tan \alpha_t}{z_{1,2}} \pm \text{inv } \alpha_t \mp \text{inv } \alpha_{tw1,2} \right\} \cdot r_{tw1,2} ;
\]

\[
S^*_{tw1,2} = \left\{ \frac{\pi \cdot m^*_{1,2}}{2 \cdot m \cdot z_{1,2}} \pm \frac{2 \cdot x_{1,2} \cdot \tan \alpha^*_t}{z_{1,2}} \pm \text{inv } \alpha^*_t \mp \text{inv } \alpha^*_{tw1,2} \right\} \cdot r_{tw1,2} ,
\]

taking into account the non-proportional division of the pitch of the output contour in the front section:

\[
m^*_{t1} + m^*_{t2} = 2 \cdot m_t ,
\]

From (6) to (10), it follows that the full thickness of the asymmetric tooth for the different possibilities for synthesis of a cylinder with a random radius is:
I possibility:

\[
S^{\Sigma} ti_{1,2} = 2.r_{1,2} \left[ \frac{\pi}{2.z_{1,2}} \pm x_{1,2} \left( \tan \alpha + \tan \alpha^* \right) \right] \pm \left( \frac{\text{inv} \alpha_{ti_{1,2}} + \text{inv} \alpha_{ti_{1,2}}^*}{2} \right) ; (11)
\]

II possibility:

\[
S^{\Sigma} ti_{1} = 2.r_{1} \left[ \frac{\pi}{4.m_{1}.z_{1}} \pm x_{1} \left( \tan \alpha + \tan \alpha^* \right) \right] \pm \left( \frac{\text{inv} \alpha_{ti_{1}} + \text{inv} \alpha_{ti_{1}}^*}{2} \right) ;
\]

\[
S^{\Sigma} ti_{2} = 2.r_{12} \left[ \frac{\pi}{4.m_{12}.z_{2}} \pm x_{2} \left( \tan \alpha + \tan \alpha^* \right) \right] \pm \left( \frac{\text{inv} \alpha_{ti_{2}} + \text{inv} \alpha_{ti_{2}}^*}{2} \right) , (12)
\]

\[m_{t_{1}} + m_{t_{2}} = 2.m_{t} .\]

The theorem of Willis is accepted as valid, ensuring constancy of the gear ratio for both parts of the profile when the direction is reversed:

\[i_{1,2} = \left[ \frac{\omega_{1}}{\omega_{2}} \right] = \left[ \frac{d_{tw_{2}}}{d_{tw_{1}}} \right] = \left[ \frac{d_{tw_{2}}^*}{d_{tw_{1}}^*} \right] = \text{const} , (13)\]

while the expression (5) takes the following form:

\[\text{inv} \alpha_{tw} + \frac{d_{tw_{1}}^*}{d_{tw_{1}}} \text{inv} \alpha_{tw}^* = \frac{2.(x_{2} \pm x_{1})}{z_{2} \pm z_{1}} \tan \alpha + \text{inv} \alpha_{t} +
\]

\[\frac{d_{tw_{1}}}{d_{tw_{1}}} \left[ \frac{2.(x_{2} \pm x_{1})}{z_{2} \pm z_{1}} \tan \alpha^* + \text{inv} \alpha_{t}^* \right] . (14)\]

Correlation (14) expresses the general form of the requirement for slack-free engagement in case of asymmetric tooth profile realized at different profile angles and composed of two independent symmetrical profile of the wheel. Actually, the wheels of the gear with asymmetric profile have one centroid because only one tool generates the tooth profiles. This requirement generates the following additional equation:
This proves that the initially accepted assumption for two initial cylinders of each wheel of the gear is false. Then correlation (14) takes the following final form for involute cylindrical gears:

\[
\text{inv } \alpha_{tw} + \text{inv } \alpha_{tw}^* = \frac{2 \left( x_2 \pm x_1 \right)}{z_2 \pm z_1} \left( \text{tg } \alpha + \text{tg } \alpha^* \right) + \text{inv } \alpha_t + \text{inv } \alpha_t^* .
\]

Therefore, the determination of the engagement angles of the asymmetrical tooth profile is done in accordance with the following system:

\[
\text{inv } \alpha_{tw} + \text{inv } \alpha_{tw}^* = \frac{2 \left( x_2 \pm x_1 \right)}{z_2 \pm z_1} \left( \text{tg } \alpha + \text{tg } \alpha^* \right) + \text{inv } \alpha_t + \text{inv } \alpha_t^* .
\]

The transcendental system (17) differs from the originally accepted system (4), which suggests that the asymmetric tooth profile exist independently outside the symmetric profiles that construct it. When the profile angles of the tool of the system (17) are equal, the correlation is obtained for determining the angle of engagement for a symmetrical profile (4). Another way of obtaining the system (17) is by subsequent summation of the left and right parts of the correlations for a symmetrical profile (4).

The given sequence for deduction of correlations from (4) to (17) is proof of the following Theorem: Reversible involute cylindrical gears with asymmetric profile of the teeth (and constant gear ratio) is realized when the inter axis distance is equal, determined for both profiles, and in case of slack-free engagement of the corresponding profiles along their initial circles.

The theorem for reversing the direction of the motion gives the necessary – slack-free engagement of the profiles and sufficient condition – equal inter axis distance, allowing synthesis of such gear. For the first time the theorem for reversing the direction of movement has been defined by authors team of the Technical University - Gabrovo in 2005 \([6,7]\), and two doctor's degrees have been defended and one habilitation thesis has been developed. Due to the long research in the field of geometric synthesis, the theorem for reversing the direction of movement is now interpreted as a basic law of the plane gearing for reverse involute cylindrical gears, while the following consequences have been summarized.

Consequences of the theorem for reversing the direction of movement

Consequence I: Equal thicknesses of the teeth of the jointly working wheels of the gear along the poloid circle are obtained within one complete pitch angle \(2 \pi / z\) – Possibility I for generation. This approach assumes that all variables in the transcendental system (17) are known, while it is solved in relation to the unknown angles of engagement \(\alpha_{tw}\) and \(\alpha_{tw}^*\):

\[
\alpha_{tw} \left( n+1 \right) = \alpha_{tw} \left( n \right) - \frac{\text{tg}^2 \alpha_{tw} \left( n \right) + \text{tg}^2 \left( \text{arccos} \left( \frac{\cos \alpha_t^*}{\cos \alpha_t}, \cos \alpha_{tw} \left( n \right) \right) \right) \pm \left( \text{arccos} \left( \frac{\cos \alpha_t^*}{\cos \alpha_t}, \cos \alpha_{tw} \left( n \right) \right) \right)}{} .
\]
Characteristic for this possibility of generation is that the sufficient condition for equality of the inter axis distance is reduced to the following form:

\[
\cos \alpha_1 \cos \alpha_{tw} = \cos \alpha^*_1 \cos \alpha^*_{tw}.
\]  

(19)

From correlation (19), it follows that the possibility for generation has the following disadvantages:

- the profile angles of the tool \( \alpha \) and \( \alpha^* \) must be known in advance;
- when a large difference between the profile angles is used, the height of the tool is decreased;
- the full form of the transcendental system (17) is not used;
- there is a wide range of data about gears with symmetric profile, which cannot be extrapolated directly onto the asymmetric profile.

All these disadvantages are eliminated by using a new option in the synthesis, which is determined by the following Consequence II: The synthesis of gears with asymmetric profile allows optimizing of the engagement of gears with symmetric profile.

Fig. 2 Difference between the initial symmetry axis OO' and the new axis OO'' for Possibility II for generation in the front section of the wheel

Based on consequence II of the theorem for reversing the direction of movement, two new approaches are offered for solving the transcendental system (17). For the geometrical interpretation of these two possibilities it is assumed that the full form of the transcendental system (17) is implemented, which implies that the asymmetry between the profiles is expressed by the basic circles and not only by the difference between the profile angles of the tool. This
implies a difference in the "initially" accepted axis \( OO' \). Fig. 2 shows the difference between the initial symmetry axis \( OO' \) and the "new" axis \( OO'' \) in the front section of the wheel. Hypothetically, we will assume that we can set the position of this new axis \( OO'' \).

It is assumed that the profile angles of the output contour \( \alpha \) and \( \alpha^* \) in the normal cross-section are known (as in possibility I). Based on the transcendental system (17) the engagement angle \( \alpha_{tw}^* \) is determined by the method of successive approximations [5] until the desired accuracy:

\[
\alpha_{tw}^{*(n+1)} = \alpha_{tw}^{*(n)} - \frac{(z_2 \pm z_1)(\arctan \alpha_{tw} + \arctan \alpha_{tw}^*) - \arctan \alpha - \arctan \alpha^*)}{\tan 2 \alpha_{tw}^* (n)}. \tag{20}
\]

The profile angle of the tool \( (\alpha_t^*) \) in the front section is determined by the sufficient condition of the theorem and correlation (15) relative to the initial axis. The general form of that other sequence of solving the system is:

\[
\begin{align*}
\arcsin \alpha_{tw}^* &= \frac{2 \left( x_2 \pm x_1 \right) \tan \alpha}{z_2 \pm z_1} + \arcsin \alpha_t \\
\frac{d_{tb}}{\cos \alpha_{tw}^*} &= \frac{d_{tb}^*}{\cos \alpha_{tw}} \\
\arcsin \alpha_{tw}^* &= \frac{2 \left( x_2 \pm x_1 \right) \left( \tan \alpha + \tan \alpha^* \right)}{z_2 \pm z_1} + \arcsin \alpha_t + \arcsin \alpha^* - \arcsin \alpha_{tw}.
\end{align*} \tag{21}
\]

Apart from this variant of solution, the transcendental system (17) offers another possibility for the same initial assumptions, but in a different sequence.

The engagement angle \( \alpha_{tw}^* \) is determined based on the ratio of the main circles, which is known in advance:

\[
\alpha_{tw}^* = \arccos \left( \frac{d_b}{d_b} \cos \alpha_{tw} \right). \tag{22}
\]

The profile angle of the tool \( \alpha_{t}^* \) is determined depending on the correlation on slack-free gearing (16) as a dependent variable until the desired accuracy:

\[
\alpha_{t(n+1)}^* = \alpha_{t(n)}^* - \frac{(z_2 \pm z_1)(\arctan \alpha_{tw} + \arctan \alpha_{tw}^* - \arctan \alpha_t - \arctan \alpha_{t(n)})}{2 \left( x_2 \pm x_1 \right) \cos 2 \alpha_{(n)}^* + (z_2 \pm z_1) \tan 2 \alpha_{(n)}^*} + \\
\frac{2 \left( x_2 \pm x_1 \right) \left( \tan \alpha + \tan \alpha_{(n)}^* \right)}{2 \left( x_2 \pm x_1 \right) \cos 2 \alpha_{(n)}^* + (z_2 \pm z_1) \tan 2 \alpha_{(n)}^*}. \tag{23}
\]
The engagement angle \( \alpha_{tw} \) relative to axis \( \text{OO}' \) is determined by the sufficient condition of the theorem and correlation (15).

\[
\begin{align*}
\sin \alpha_{tw} &= \frac{2 \left( x_2 \pm x_1 \right) \tan \alpha}{z_2 \pm z_1} + \sin \alpha_t \\
\cos \alpha_{tw} &= \frac{d_{ib}^*}{d_{ib}} \cos \alpha_{tw}
\end{align*}
\]

(24)

Such optimization of the asymmetric profile based on the theorem for reversing the direction of movement has been implemented for and output involute cylindrical gear with outer engagement and the following parameters of the output contour: \( m=1 \text{mm}, \alpha=\alpha^*=20^\circ, h^*_{a}=1, h^*_{f}=1,25, \) displacement of the contour \( x_1=x_2=0,5, \) number of teeth of the wheel 1 of the gear \( z_1=20, u=1.5, \beta=0 \) and open profile milling method. The data about the main qualitative indicators and comparison with the output symmetric profile are given in Table 1.

As a basic parameter for the optimization (Table 1) the criterion of maximum front overlap ratio \( \varepsilon_{\alpha}=\max \) was used. One optimal variant has been determined without clipping at integer value of \( \alpha^* \). This possibility of solving obtains increase of the front overlap ratio \( \varepsilon_{\alpha}=1,33 \) for symmetric profile up to \( \varepsilon^*_{\alpha}=1,44 \) for asymmetric profile – possibility II.

Table 1 Qualitative indicators of symmetric and asymmetric profiles generated through Possibility II

<table>
<thead>
<tr>
<th>Gear</th>
<th>Angles of the contour for optimized profile</th>
<th>Thicknesses of the teeth along the crown circles</th>
<th>Overlap ratio</th>
<th>Specific slipping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric</td>
<td>( \alpha^*=20^\circ )</td>
<td>( \text{Sa}_1=0.62 ) \text{Sa}_2=0.70</td>
<td>( \varepsilon_{\alpha}=\varepsilon^*_{\alpha}=1,33 )</td>
<td>(</td>
</tr>
<tr>
<td>Asymmetric II</td>
<td>( \alpha^*=15^\circ )</td>
<td>( \text{Sa}_1=0.67 ) \text{Sa}_2=0.76</td>
<td>( \varepsilon^*_{\alpha}=1,44 )</td>
<td>(</td>
</tr>
</tbody>
</table>

This proposed new possibility of generation has the following advantages:

- uses the available data for a symmetric profile;
- allows synthesis of asymmetric profile, where the slack-free engagement is achieved relative to an axis different from the initial one;
- the limitation of the tool is eliminated, while the profile angle \( \alpha^* \) varies within the accepted proportionality between the two basic (generating) circles.

From correlation (21) and (24), it follows that possibility II for synthesis does not change the necessary and sufficient condition for reversing the direction of movement, according to the theorem for reversing the direction. Therefore, the involutes generated through possibility I are equidistant to those generated through possibility II.
Compared to possibility I for synthesis, possibility II offers another sequence for solving the transcendental system (17), and namely a conversion of the classic theory of tooth engagement and creation of new gears hitherto unknown qualitative and strength parameters.

CONCLUSIONS

1. Based on the classical theory of tooth engagement, the following are defined: axiom of the tool engagement in case of asymmetry of the tooth profile, basic parameters of the output contour, necessary for the production of wheels by the method of centroid wrapping and the initial geometric conditions for synthesis of conjugated wheels and gears.

2. A theorem has been defined and proven about the reversing of the direction of movement of involute cylindrical gears with asymmetric profile of the teeth, through which their geometric synthesis is possible.

3. The existence of two conditions was proven for reversing the direction of the involute cylindrical toothed gears with asymmetric profile, providing slack-free engagement and equality of the inter axis distance.

4. Two consequences have been defined of the theorem for reversing the direction of movement, allowing different starting poloids of the gear wheels, as well as various improved qualitative indicators of the tooth engagement, which it is impossible if the teeth have symmetrical profile.

5. The existence was proven of two possibilities for generation of the asymmetric tooth profile, giving different qualitative and strength parameters at equal initial variables of the synthesis, which has been shown by example of optimization of a gear with asymmetric profile of the teeth according to the criterion of maximum front overlap ratio $\varepsilon_{\alpha} = \text{max}$.

6. The axiom for tool engagement of involute cylindrical gears with asymmetric profile of the teeth, the theorem for reversing the direction of movement and the resulting consequences are the basic law on gearing of a new generation of toothed gears by using the classic approach of geometric synthesis with partially explored to this moment indicators of quality, strength and precision.

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