A New Approach for Graph Coloring

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ABSTRACT

This paper describes about the frequency assignment problem of graph coloring. An efficient algorithm has been proposed to provide least frequencies to the cells of the given area. The proposed algorithm is applicable to all types of graphs. The algorithm searches for the neighbors of the first vertex colored and then looks for the further coloring based on the algorithm proposed. We compare our results with the base frequency allocation algorithm in terms of time utilization to cover all the graphs and the frequencies allocated to the cells. Experiments have been conducted on the previously used graphs both standard and random so as to easily demonstrate the results and the effectiveness of the algorithm applied.

Keywords: Graph Coloring, Neighbors of a Vertex, Combinational Optimization.

INTRODUCTION

In computer science applications graph imaginary ideas are highly utilized. Mainly in research areas of computer science such image segmentation, data mining, image capturing, clustering, networking etc., For example a data structure can be considered in the form of tree which in turn utilized vertices and edges. Using the concepts of graph correspondingly modeling of network topologies can be done. In the same way the most important theory of graph coloring is utilized in resource allotment, scheduling. Also, paths, walks and circuits in graph theory are used in incredible applications say database design concepts, resource networking, travelling salesman problem. This leads to the expansion of new algorithms and new theorems that can be used in incredible applications.

A graph is a pair G = (V, E) of units satisfying E ⊆ V × V . The factors of V are the vertices of the graph G, the elements of E are its edges. In literature, graphs are also known as simple graphs; vertices are referred to as nodes or points; edges are called lines or hyperlinks. The list of alternatives is lengthy (but still finite). A pair {u, v} is usually written certainly as uv; note that then uv = vu. For you to simplify notations, we additionally write v ∈ G and e ∈ G instead of v ∈ VG and e ∈ EG.

History of Graph theory:

The beginning of graph principle started out with the hassle of Koinsberbridge, in 1735. This problem result in the idea of Eulerian Graph. Euler studied the trouble of Koinsberg bridge and constructed a structure to remedy the problem known as Eulerian graph. In 1840, A.F Mobius gave the concept of complete graph and bipartite graph and Kuratowski proved to they are planar through recreational problems. The concept of tree, (a linked graph with out cycles[1]) become applied by way of Gustav Kirchhoff in 1845, and he hired graph theoretical thoughts in the calculation of currents in electric networks or circuits. In 1852, Thomas Gutherie found the well-known 4 color problem. Then in 1856, Thomas. P. Kirkman and William R.Hamilton studied cycles on polyhydra and invented the idea referred to as Hamiltonian graph by studying journeys that visited certain web sites precisely once. In 1913, H.Dudeney mentioned a puzzle trouble. Eventhough the 4 color hassle was invented it became solved most effective after a century by Kenneth Appel and Wolfgang Haken. This time is precise as the beginning of Graph theory.

Caley studied precise analytical forms from differential calculus to examine the trees. This had many implications in theoretical chemistry. This lead to the invention of enumerative graph principle. Any how the term “Graph” become delivered by means of Sylvester in 1878 where he drew an analogy among “Quantic invariants” and covariants of algebra and molecular diagrams. In 1941, Ramsey worked on colors which lead to the identification of every other department of graph principle referred to as extremel graph theory. In 1969, the 4 colour problem become solved the usage of computer systems by Heinrich. The examine of asymptotic graph connectivity gave upward thrust to random graph principle.[1].
Applications of Graph theory:

Graph theoretical ideas are broadly used to study and version various applications, in different regions. They include, examine of molecules, creation of bonds in chemistry and take a look at of atoms. Similarly, graph concept is utilized in sociology for example to measure actors status or to discover diffusion mechanisms. Graph concept is utilized in biology and conservation efforts where a vertex represents areas where positive species exist and the edges represent migration course or movement among the areas. This data is important when searching at breeding styles or monitoring the spread of disease, parasites and to observe the impact of migration that have an effect on other species. Graph theoretical concepts are extensively utilized in Operations studies. As an example, the travelling salesman trouble, the shortest spanning tree in a weighted graph, obtaining an most fulfilling match of jobs and guys and finding the shortest course between two vertices in a graph. It is also used in modeling shipping networks, activity networks and concept of video games. The community hobby is used to solve massive range of combinatorial problems. The maximum popular and a success packages of networks in OR is the planning and scheduling of massive complicated tasks. The great widely recognized troubles are CPM (Critical Path Method) and PERT(Project Evaluation Review Technique). Next, game idea is carried out to the troubles in engineering, economics and conflict technology to discover most appropriate manner to carry out sure tasks in aggressive environments. To represent the approach of finite game a digraph is used. Right here, the vertices represent the positions and the edges constitute the moves.

Graph Coloring:

Graph coloring is one of the most critical ideas in graph principle and is used in many actual time programs in computer science. Numerous coloring techniques are to be had and may be used on requirement basis. The right coloring of a graph is the coloring of the vertices and edges with minimal quantity of colors such that no two vertices have to have the same colour. The minimal quantity of colors is called as the chromatic number and the graph is referred to as well colored graph.[1].

Frequency Assignment Problem

The frequency assignment problem has two fundamental aspects:

(i) A set of wi-fi communication connections should be assigned frequencies such that information transmission between the transmitter and receiver for each connection is feasible. The frequencies ought to be selected from a given set that can differ among connections. Note that much traffic is bidirectional, so that during truth two frequencies must be selected, one for each path.

(ii) The frequencies assigned to two connections may also incur interference ensuing in loss of quality of the signal. Two conditions have to be fulfilled on the way to have interference of two signals:

(a) The two frequencies have to be near on the electromagnetic band (Doppler effects) or (close to) harmonics of one another. The latter effect seems to be inadequate, however, since the frequency bands from which we can select are generally so small that they do not contain harmonics.

(b) The connections should be geographically close to each other. The signals that could interfere ought to have a similar level of energy at the location where they may disturb each other.

The significance of the frequency assignment problem (FAP) for efficient use of the radio spectrum and minimization of interference is now well identified. The general public of the early work on this problem targeting on the minimum span problem [2]-[3]. In this modification of the FAP, constraints are precise, which, if satisfied, should cause suited interference. It is far then essential to minimize the spectrum used, measured as the distinction among the finest and smallest assigned frequencies. It is now increasingly diagnosed that the fixed spectrum frequency assignment problem (FS-FAP) is of better significance to network operators. In this modification of the FAP, the available spectrum to the operator is known in advance and is necessary to reduce a few degree of interference.

The FAP is a generalization of the well recognized graph colouring problem [4], which is the problem of finding a colouring of a graph in order that the quantity of used colourations is minimized, the condition is that no two adjacent vertices have the same colors: as such FAP is a NP-hard problem [5]-[6]. Such issues require the usage of extraordinarily time-consuming algorithms to obtain precise solutions. It is, consequently, important to use more time-efficient algorithms that, however, cannot guarantee most effective solutions. Distinct lower bounds on the most effective solution cost for the frequency assignment problem have been proposed [7]-[8].
These techniques are useful both in assessing the high-quality of approximate solutions and in restricting the search for finest assignments and are generally derived from graph-theoretic approaches, which adapt techniques in the beginning evolved for the colouring problem.

Different approaches were used, such as tabu search [11], genetic algorithms [13], neural networks [5], dynamic programming [14], ant colony optimization [15].

The rest of this paper is organized as follows. Section II presents the Related work. Section III discusses the performance evaluation and experimental results. Finally, Section IV concludes the paper.

RELATED WORK

Luby(1985) offered a randomized algorithm which require a linear quantity of processors. the main idea is to obtain for every vertex a neighborhood general order or a nearby election which breaks the nearby symmetry and then every vertex can determine locally whether to joins the MIS or not.

Linial(1992) confirmed a identical lower bound for time required for 3 coloring of a n-cycle and proved that there is no steady time dispersed algorithm.

Any other remarkable work from the sector of allotted algorithms designed for graph coloring is the distributed largest First(DLF) set of rules introduced in Kubale and Kuszner (2002). DLF is an development over an set of rules for randomly ordering the nodes and the usage of this order to allow nodes to choose colours below the constraint that shades which have been previously assigned to neighbors cannot be used. DLF differs from the authentic set of rules version inside the order of the nodes. In DLF, the higher degree of a node the sooner the node will be able to pick out a coloration. The random order is most effective used to break ties among nodes that have the equal degree. DLF carried out higher results than the original set of rules on random graphs.

Madhav V. Marathe[2004] have done an empirical take a look at of a distributed an empirical examine of a dispersed edge coloring. They studied numerous classes of graphs viz. tress, bipartite graphs, cliques, hypercubes and random graphs. one of the goal of their study became to see how the usage of some more colors impacts the performance and suggested that if 5%-15% more colorations are used then the performance is considerably better.

One of the most popular works become provided by Finocchi (2005). The authors introduced 3 versions of a distributed set of rules and studied its behavior under various situations. The authors taken into consideration both the trouble of obtaining O(deg(v)+1) colorings in as few communique rounds as feasible, as well as the trouble of generating the satisfactory viable shades with none limit on the numbers of conversation rounds. Right here,Δ(G) refers back to the maximum degree of a graph. The authors provide significant experimental outcomes for both instances. Most in their experimentation is based totally on random graphs, which are not publicly to be had, however, in addition they provided results on a well-known set of publicly available instances from the DIMACS undertaking (Center for Discrete Mathematics and Theoretical Computer Science 2006).

In 1965, Zadeh introduced the belief of fuzzy set that is characterized by way of a membership function which assigns to each object a grade of membership which degrees from 0 to 1. The first definition of fuzzy graph became introduced by Kaufmann (1973), based totally on Zadeh’s fuzzy members of the family (1971). As defined in[5] fuzzy graphs may be defined through considering fuzzy set of crisp graphs or fuzzy aspect set with crisp vertex set or fuzzy vertex set with crisp area set or fuzzy vertex and part set or crisp vertices and crisp graph with fuzzy weights or edges with fuzzy connectivity. In this paper we recall the fuzzy graphs with crisp vertex set and fuzzy edge set.

LARGEST DEGREE ORDERING:- This is one of the heuristic sequential algorithms to solve the graph coloring problem. This set of rules comes under the class of the degree based ordering method. The fundamental idea is to choose the following vertex to be colored. This set of rules (Dr. H Al-Omari et al., 2006) chooses a vertex with highest wide variety of neighbor. Suppose a vertex have 5 neighbors and different vertex have 6 neighbors, then we can select the one with 6 neighbors and coloration it first. This algorithm might be used in the last when the exertion of cuckoo search is entire.

The cuckoo search algorithm is a recently evolved meta-heuristic optimization set of rules, that is appropriate for fixing optimization issues. To address this problem, a right method for tuning the cuckoo search parameters is presented. subsequently, we are able to get the improved overall performance of the cuckoo search. This optimization algorithm (RajabiounRamin, 2011) shows the life of a bird circle of relatives, referred to as Cuckoo. The basic motivating aspect at
the back of this algorithm is the special lifestyle and egg laying conduct of these birds. Cuckoo Optimization algorithm (COA) begins with an initial population. The cuckoo population, in distinctive societies, is in kinds: mature cuckoos and eggs. The effort to live on among cuckoos constitutes the idea of Cuckoo Optimization set of rules. At some stage in the survival competition some of the cuckoos or their eggs, death. The survived cuckoo societies immigrate to a higher environment and begin reproducing and laying eggs. Cuckoos’ survival attempt with any luck converges to a nation that there is just one cuckoo society, all with the identical income values. Application of the proposed algorithm to a few benchmark functions and a actual trouble has tested its functionality to deal with tough optimization problems.

The pseudo code for cuckoo optimization algorithm is as follows:

1. Initialize cuckoo habitat with some random points on the profit function.
2. Commit a few eggs to each cuckoo.
3. Outline ELR for every cuckoo.
4. Let cuckoos to put eggs interior their corresponding ELR.
5. Kill the ones eggs that are identified by host birds.
6. Let eggs hatch and chicken developed.
7. Examine the habitat of each newly grown cuckoo.
8. Limit cuckoos maximum number in environment and kill individuals who stay in worst habitat.
9. Cluster cuckoos and discover excellent institute and choose correct habitat.
10. Let new cuckoo population immigrate in the direction of goal habitat.
11. If stop condition is satisfied stop, else go to step 2.

This Mantegna algorithm (Abdul et al., 2011) is an necessary a part of the cuckoo search and is needed in producing the random numbers consistent with a symmetric levy strong distribution. In this set of rules, the distribution’s parameter $\alpha$ lies inside the range of $[0.3, 1.99]$ and $c > 0$. Right here $n$ is the number of iteration and the variety of random factors to be produced. If an input parameter is outside the valid range, an errors message is displayed and the production consists of an array of NaNs (not a number).

**Proposed Work**

The proposed work in graph theory can be done in various fields. We have choosen frequency assignment problem as one of the most important application in graph coloring field. The graph theory has various applications and frequency assignment plays an important role.

We have considered graphs for various cells and nodes representing the frequency allocation in mobile networks. The problem of frequency assignment is to provide wireless communication frequencies from limited spectral resources while keeping to a minimum interference suffered by those wishing to communicate in a given radio communication network. There are two principal types of FAP studied so far. The first one is fixed frequency assignment problem (F-FAP) which is a static model where the set of connections remains stable over time. The other one is called dynamic frequency assignment problem (D-FAP).

**Proposed Algorithm**

The proposed algorithm is being shown here with the help of various basic new ideas that can work on performance and frequency assignment to the cells in a mobile network. The cell network can be shown in three types of networks. These are- vertex coloring, edge coloring and region coloring. The proposed work has been done on vertex coloring in which vertices are connected with the help of edges which connects the nodes i.e. the cells of the given area. The flowchart of the process is shown below:
Here, the flowchart gives a full process view of the processes taking place in the order assigned. The frequency assignment can be correlated to the graph coloring problem in such a way given below:-

1. The frequency is considered as the color allocated to the node. The node is known as the cell which is present in an area for capturing the frequency. The nodes/cells can be given a particular frequency.
2. The basic idea behind the proposed algorithm is to reduce the time consumption to set frequencies for the nodes/cells of the given area and also if possible, then, to reduce the frequency as much as possible.

**Pseudo-code for the given algorithm:**

Input: frequency fr;
      Cell c;
      Area of frequency assignment is denoted as area;
Output: Colored graph/area generation.

Algorithm:
1. Load data.
2. Create graph.
3. Color first cell with 1st frequency
   \[ V(1) = 1 \]
4. For i = 1 to N

Fig. 1 Flowchart of the proposed algorithm
If ( V(i) = 1 )
  Find the neighbour cells
  For j = 1 : length (neighbours)
    Traverse graph & find all non_visited neighbour cells
  For k = 1 to length ( no._of_frequencies)
    If ( non_visited cell = = frequency )
      Provide cell a frequency & set it visited
End

5. View the graph area.

Experimental Results

The experimental results of the proposed theory of graph are presented in this section. We have taken an example for an area having cells being represented as the nodes. Initially, the cells are not colored and they have been connected to each other via edges. These edges are represented and connected to each cell on the basis of the connection of a given area. The whole view gives an idea about a scenario that can be given frequencies based on the connectivity.

After the application of the algorithm, the frequency assignment of the graph given here can be allocated based on the no.of frequencies. The frequencies should be minimum for the no. of cells in an area. The graph drawn here works on the principle of the proposed algorithm. The basic idea is that no two adjacent nodes can have same colors. We can see from the example we have shown that the minimum number of colors used in our graph are 3 i.e. the least frequency required to cover an area for cells is 3.
RESULTS ANALYSIS

The results are calculated on few parameters. These parameters can be explained as follows:-

1. **Time utilization** - the time consumed by the proposed algorithm is considered. The time taken by the proposed algorithm should be less than the base algorithm.

   \[ \text{Time} = \text{total time utilized to traverse all nodes and providing color to them.} \]

2. **Frequency assignment** - minimum frequency should be allotted to the cells connected in an area. The frequency allotted to the cells is given by applying algorithm.

   \[ \text{Frequency assignment} = \text{net minimum frequencies allotted to the nodes in a graph} \]

We have performed experiments on existing base algorithm and our proposed new algorithm as per the paper described. For simulation purpose, we have used MATLAB R2013a version for getting the appropriate results as shown in various figures. All experiments were performed on Dell workstation with 4 GB RAM and 32-bit operating system, windows 8. We first applied the base algorithm to find the total frequencies allotted to them. Next, we perform all the possible calculations on various graphs taking different nodes. The new database with all the values are stored in Microsoft Office Excel 2013.

I. **CPU time Utilization**

We have performed the calculations on both the algorithms. The time utilized by both the algorithms have been calculated and is compared with the help of graph. The comparison graph has been shown below:-

![Comparison graph](image)

The above graph shown that our new proposed algorithm takes less time to traverse and color the cells as compared to the base algorithm. Thus, it is better in one aspect.

II. **Frequencies assigned**

<table>
<thead>
<tr>
<th>Number of nodes</th>
<th>Frequencies Allocated</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
</tr>
</tbody>
</table>
CONCLUSION

In this paper, we have devised a new graph coloring based algorithm for cellular coloring problem. One of the main fields in graph coloring is frequency allocation problem to the various nodes. The frequency allocation of the cells is allocated in a proper way. The time utilization for the proposed algorithm have been far less than the previously designed algorithm. The graph coloring for frequency assignment thus, works on its application and makes it a new and a better field for research. Results and experiments illustrate that the scheme presented in proposed algorithm can be used for any type of graph i.e. there is no boundation of graph types. The algorithm is more user-friendly and easily understandable.

REFERENCES

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