

Start up Cost constraint Optimization using Lagrangian Algorithm for Unit Schedule in Electrical Power System

Navpreet Singh Tung¹, Ashutosh Bhadoria², Anant Bhardwaj³

¹M.Tech Scholar, Electrical Engg. (Student Member IEEE), Lovely Professional University, India

³M.Tech Scholar, Electronics and Communication Engg. Lovely Professional University, India

²Asst. Professor, Electrical Engg., Lovely Professional University, India

ABSTRACT

Electricity companies typically possess numerous units and they need to commit units because electricity cannot be stored in a large-scale system and demand is a random variable process fluctuating with the time of the day and the day of the week. A problem that must be frequently resolved by a electricity utility is to economically determine a schedule of what units will be used to meet the forecasted demand, and satisfy operating constraints such as start up cost, over a short time horizon. This problem is commonly referred to as the unit commitment (UC) problem. Lagrangian algorithm is one of the technique based on equal IC of fuel input for the units in operation. It is helpful for the optimum load sharing among units, with satisfying constraints under different environment. Simulation algorithm is prepared in this paper, keeping start up cost constraint optimization and simulation is done with Matlab for standard set of Units. Optimized IC and load sharing values are extracted sharing different start up cost. Different IC values are extracted for different load demands.

Keywords: Unit Commitment (UC); Economic Dispatch (ED), Lagrangian Multiplier (LM), Incremental cost (IC), Load Schedule (LS).

I. INTRODUCTION

The optimal system operation, in general involves the account of economic operation, system security, emissions at certain fossil-fuel plants, optimal releases of water at hydrogenation etc. All these considerations may make for conflicting requirement and usually a compromise has to be made for optimal system operation. Here, economy of operation also called the economic dispatch problem. The main aim in the Economic dispatch problem is to minimize the total cost of generating real power (production cost) at various stations while satisfying the loads and the losses in the transmission links. Normally, hydro plants operates in conjunction with thermal plants. While there is negligible operating cost at a hydro plant, there is a limitation of availability of water over the period of time which must be used to save maximum fuel at the thermal plants. In load flow problems, two variables are specified at each bus and the solution is then obtained for the remaining variables. The specified variables are real and reactive powers at PQ buses, real powers and voltage magnitudes at PV buses and voltage magnitude and angle at the slack bus.

The additional variables to be specified for load flow solution are the tap settings of regulating transformers. If the specified variables are allowed to vary in a region constrained by practical consideration (upper and lower limit on active and reactive generations bus ,voltage limits, and range of transformer tap settings) there results an infinite number of load flow solutions, each pertaining to one set of values of specified variables. The best choice in some sense of the values of specified variables leads to the best load flow solution. Economy of operation is naturally predominant in determining allocation of generation to each station for various system load levels. The first problem in power system is called the 'Unit Commitment (UC) problem and the second is called the 'Load Scheduling' (LS) problem. One must first solve the UC problem before proceeding with the LS problem. We are concerned ourselves with an existing installation, so that the economic considerations are that of operating (running) cost and not the capital outlay.

LM has been successfully applied to the complex UC problem including various hard constraints (e.g. ramp rate constraints, minimum up and down time, etc.)[15]. However, ramping constraints in UC problem required

enlarging state spaces dramatically for dynamic programming to solve each unit sub-problem[16]. The total number of states was the sum of number of down states, number of ramp up states, number of up states, and number of ramp down states [17].M. Bavafa et. al proposed a hybrid Lagrangian relaxation with evolutionary programming and quadratic programming (LREQP) for ramp rate constrained unit commitment (RUC) problem [15].Xiaohong Guan et. al proposed that Lagrangian relaxation (LR) is one of the most successful approaches [18-20]. One of the most obvious advantages of the Lagrangian relaxation method is its quantitative measure of the solution quality since the cost of the dual function is a lower bound on the cost of the primal problem. For UC problems, the duality gap, the relative difference between the feasible cost and the dual cost is rather small, often with (1-2)% . This accuracy was considered sufficient for industrial applications before the emergence of wholesale competitive energy markets [21-24].Weeraya Poommalee et. al (2008) obtained the unit commitment considering security-constrained optimal power flow (UC-SCOPF) by using Lagrangian relaxation with genetic algorithm (LRGA) [25].

II. OPTIMAL OPERATION OF GENERATING UNITS

- **Mathematical formulation of Generator Operating Cost**

The major component of generator operating cost is the fuel input/hour, while maintenance contributes only to a small extent. The fuel cost is meaningful in the case of thermal and nuclear stations, but for hydro stations where the energy storage is 'apparently free', the operating cost as such is not meaningful. We concentrate on fuel fired stations.

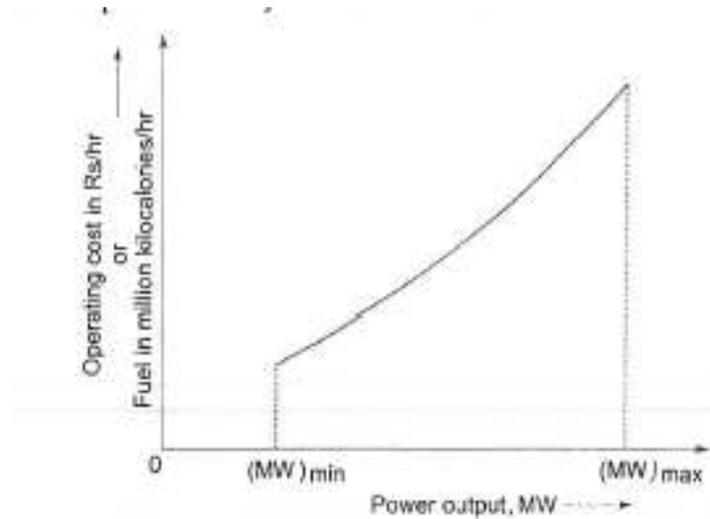


Fig.1 Input-output curve of Generating unit

The input-output curve of a unit can be expressed in a million kilocalories per hour or directly in terms of rupees per hour versus output in megawatts. The cost curve can be determined experimentally. A typical curve is shown in Fig. 1 where (MW)min is the minimum loading limit below which it is uneconomical (or may be technically infeasible) to operate the unit and (MW)max is the maximum output limit. The input-output curve has discontinuities at steam valve openings which have not been indicated in the figure. By fitting a suitable degree polynomial, an analytical expression for operating cost can be written as

$$C_i(P_{Gi}) \quad \text{Rs/Hour at output } P_{Gi}$$

where the suffix i stands for the unit number. It generally suffices to fit a second degree polynomial

$$C_i(P_{Gi}) = 1/2a_i P_{Gi}^2 + b_i P_{Gi} + d_i \quad \text{Rs/Hour} \text{-----(1)}$$

The slope of the cost curve is $dC_i/d P_{Gi}$, called the incremental fuel cost(IC) and is expressed in units of rupees per megawatt hour (Rs/MWh).A typical plot of incremental fuel cost versus power output is shown in Fig.2.If the cost curve is approximated as a quadratic as in Eq. (1), we have

$$(IC)_i = a_i P_{gi} + b_i \text{-----(2)}$$

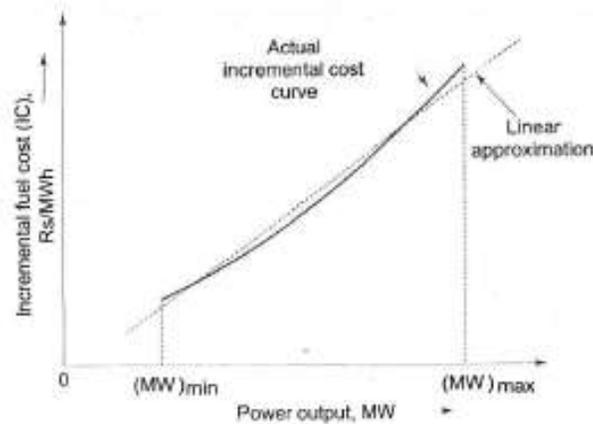


Fig.2 Incremental fuel cost versus power output for the unit whose input-output curve is shown in Fig.1

i.e. a linear relationship. For better accuracy incremental fuel cost may be expressed by a number of short line segments (piecewise linearization). Alternatively, we can fit a polynomial of suitable degree to represent IC curve in the inverse form-

$$P_{Gi} = A_i + B_i(IC_i) + C_i(IC_i)^2 \dots (3)$$

• **Formulation of Optimal Operation**

Let us assume that it is known a priori which generators are to run to meet a particular load demand on the station.

$$\sum P_{Gi, \max} \geq P_d \dots (4)$$

where $P_{Gi, \max}$ is the rated real power capacity of the i th generator and P_d is the total power demand on the station. Further, the load on each generator is to be constrained within lower and upper limits, i.e

$$P_{Gi, \min} \leq P_{Gi} \leq P_{Gi, \max} \text{ where } i=1, 2, \dots, k \dots (5)$$

Considerations of spinning reserve require that

$$\sum P_{Gi, \max} > P_d \text{ by proper margin} \dots (6)$$

Since the operating cost is insensitive to reactive loading of a generator, the manner in which the reactive load of the station is shared among various online generators does not affect the operating economy. The question that has now to be answered is: 'What is the optimal manner in which the load demand P_d must be shared by the generators on the bus keeping SC constraint optimized?' This is answered by minimizing the operating cost

$$C = \sum C_i(P_{Gi}) + SC \text{ under the equality constraint of meeting the load demand i.e.}$$

$$\sum P_{Gi} - P_d = 0 \dots (7)$$

where k is the number of generators on the bus.

Further, the loading of each generator is constrained by the inequality constraint of Eq. (3). Since $C_i(P_{Gi})$ is non-linear and C , is independent of P_{Gj} , this is a separable non-linear programming problem.

• **Lagrangian Algorithm**

$$1) \mu = \sum C_i(P_{Gi}) - \lambda (\sum P_{Gi} - P_d)$$

Where λ is langrangian multiplier

2) Minimization is achieved by

$$d\mu / dP_{Gi} = 0; \text{ or}$$

$$dC_i / dP_{Gi} = \lambda; i=1, 2, \dots, k$$

where dC_i/dP_{Gi} is the incremental cost of i th generator (Units=Rs/MWh), a function of generator loading P_{Gi} .
 $dC_1/dP_{G1} = dC_2/dP_{G2} \dots dC_i/dP_{Gi} = \lambda \dots (8)$

i.e. the optimal loading of generators corresponds to the equal incremental cost point of all the generators.
 Eq.(8) is called the co-ordinate equations numbering k are solved simultaneously with the load demand equation(4) to yield a solution for the lagrangian multiplier λ and the optimal loading of k generators.

• **Flow Chart for Computer Simulation**

- 1) Choose trial value of λ i.e. $IC = (\lambda)^0$
- 2) Solve for P_{Gi} from eq.(4)
- 3) If $|\sum P_{Gi} - P_d| < \epsilon$ (a specified value) the optimal solution is reached.
 Otherwise,
- 4) Increment λ by $\Delta(\lambda)$ if $|\sum P_{Gi} - P_d| < \epsilon$ or decrement λ by $\Delta(\lambda)$, If $|\sum P_{Gi} - P_d| > 0$ and repeat from step 2.. This step is possible because P_{Gi} is monotonically increasing function of (λ)

• **Effect of equality constraint**

As λ is increased or decreased in iterative process, if a particular generator loading reaches its P_{Gimax} or P_{Gimin} , its loading from now on is held fixed at this value and the balance load is shared between remaining units based on equal λ basis.

III. EXPERIMENTAL ANALYSIS AND SIMULATION RESULTS

• **Assumptions**

- Units are in operation all the time
- All the losses are neglected
- Only few constraints are considered for simplicity generated. Other constraints are relaxed.

• **Parameter setup**

Incremental Fuel Cost equations for plant in Rs/MWh for 2 units-

- $dC_1/dP_{G1} = 0.20P_{G1} + 40$
- $dC_2/dP_{G2} = 0.25P_{G2} + 30$
- SC=10
- N=2
- $P_d = 231.5$
- Tolerance=1
- Initial Lamda Value(λ)=20
- PGmax=[125 125]
- PGmin=[20 20]
- Constants A=[0.20 0.25]
- B=[40 30]

• **Experimental Values**

Table 1. Committed Schedule combinations of units

U1	U2
0	0
1	0
0	1
1	1

0-OFF, 1-ON, U1-Unit 1, U2-Unit 2

Table.2 Experimental Variation of Incremental Cost with Load demand and distribution of loads between generating units

Lamda(Rs/MWh)	Load		
	MW	U1(MW)	U2(MW)
35	40	20	20
49	60	20	39
49	90	27	62
49	120	45	76
52	150	61	89
59	200	100	100
59	210	95	116
60	220	100	120
63	230	105	124
63	240	115	125
65	250	125	125

In table 2.,optimized value of Incremental cost is estimated under different load demands with subjected to optimum load distribution between two units. As the load demand tends to increase, unit 2 is subjected to more load bearing capacity.Unit 2 achieves its maximum limit prior to unit 1 at load demand of 240 MW or more.Incremental cost tends to vary in step size as load demand increases as seen in Fig 4.

Table.3 Experimental Variation of Total Cost with equal IC and equal distribution of loads between generating units including SC

Total Cost with SC(Rs/hr)	Equal Load Sharing Cost(Rs/hr) with SC
89	89
91.5	93.5
100.9	100.25
108	107
116.05	113.75
125	125
128	127.25
130	129.5
132	131.75
136	134
136.5	136.5

In table 3, experimental values shows that upto 90 MW load demand,IC rule for sharing of load results in minimum total cost including SC.Beyond 90 MW,equal sharing of load results in total minimum cost.Both Units U1 and U2 should share equal load for the total cost to be minimized.Here SC is fixed at 10 MW.With little variation in SC for two units,sharing of load may be different for the cost to be minimized.

- **Test Result**

For load demand upto 90 MW,LM rule should be followed .Beyond 90 MW,equal sharing of load should be done to minimize the total cost.

- **Graphical representation of Parameters**

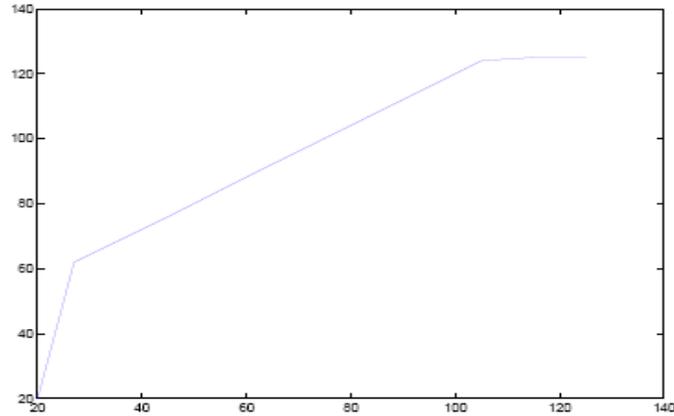


Fig 3. Distribution of Loads between U1 and U2

In fig.3, U2 tends to bear more load than U1 as in starting ,incremental cost for U1 is high. To reach that level of IC,U2 has to bear more load.When U1 is at 25MW,U2 is at 60 MW.U2 continues to bear more load as the load demand increases as per the slope of between interval (25-100MW for U1).

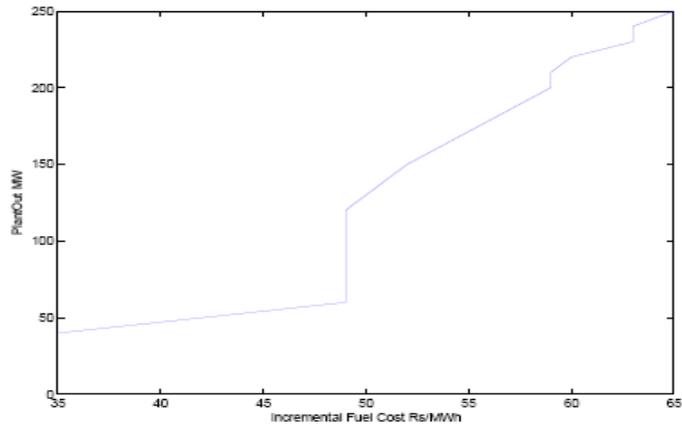


Fig 4.Variation of Incremental Cost(IC) vs Plant output

In Fig.4,IC tends to vary sharply as per the slope when load demand is low.When load demand increases,IC increases in step size as given in table 2.

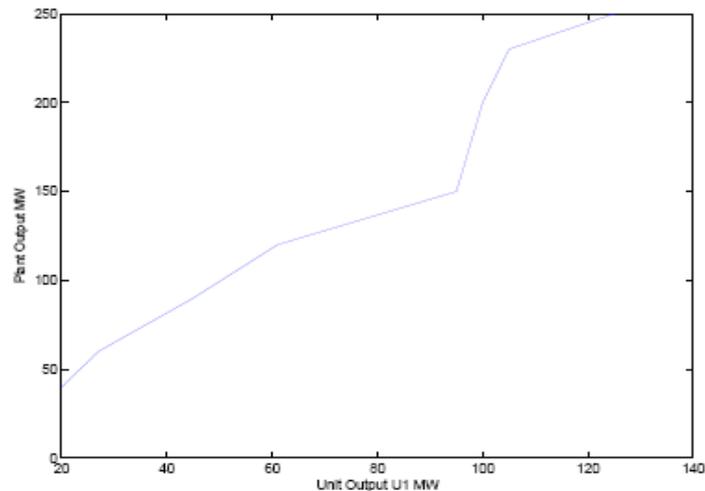


Fig 5. Variation of Plant output with Unit output (U1)

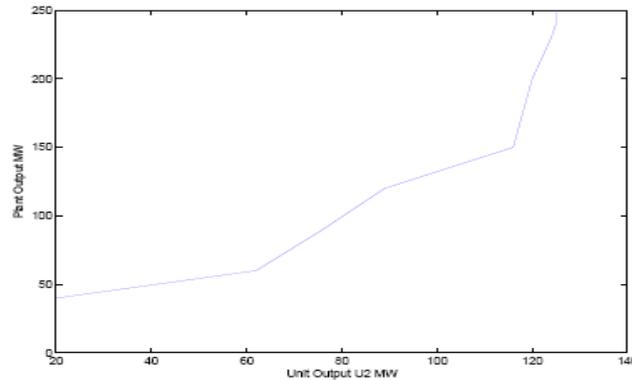


Fig 6. Variation of Plant output with Unit output (U2)

In fig.5 and 6,U2 is having less slope as compared to U1 with variation in load demand.U1 is increasing linearly till near to its maximum limit.U2 experienced sharp rise in slope when power demand jumps above 80 MW as given in table 2 till the optimized sharing of load above 150MW.

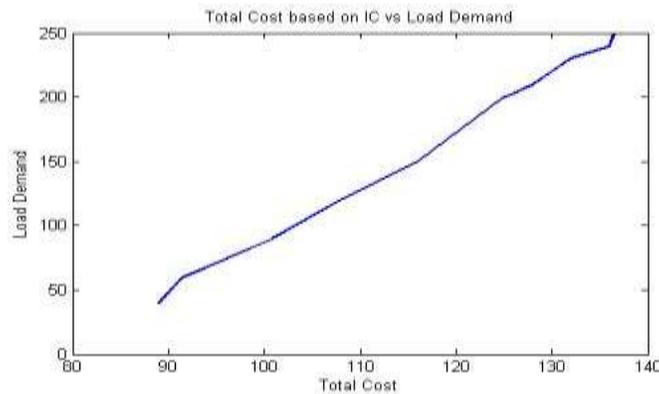


Fig 7. Variation of Total cost based on IC with Load Demand

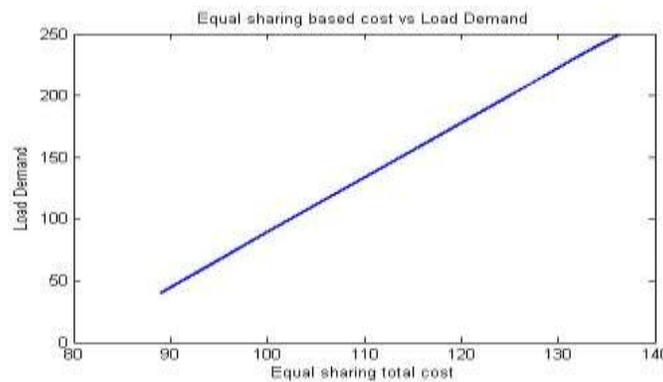


Fig 8. Variation of Total cost based on equal sharing of load with Load Demand

CONCLUSION

In this paper, the optimized value of total cost including start up cost is analyzed for various load demands. The experimental results shows that IC rule can work upto 90 MW and beyond 90 MW, equal sharing of load works to minimize the total cost. IC value is extracted using LM.A simple computer model is prepared for the computer simulation using Matlab. Load sharing at different load demand is reflected in simulation studies with the computation of optimized IC value, total cost including SC. This model can be extended to n no. of units and at

different load demands. Different SCs for different loads can be incorporated to optimize the total cost. Combinations of Thermal and Hydro plant can be used as per the load demand and the impact of start up cost. With small step size of $\Delta\lambda$, more accuracy can be observed although computation time and memory are involved. Future work incorporates artificial intelligence including more constraints in this technique.

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