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Consistency of Plane Symmetric Models in Modified Gravitational Theory

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ABSTRACT

To understand the late time acceleration of the universe, the Modified Gravitational Theories received more attention lately. Essentially its view point is modifies the geometric components. Among the various extension to the theory of gravity, the higher order curvature invariant are specially included, specifically the class of f(R,T) theories have received many acknowledgments. The motive of this work is to explore the study of Plane Symmetric Cosmological Model in modified theory of gravitation f(R,T) with perfect fluid. Discuss about its consistency, by solving the field equation off (R,T) with specific choice of function. The solution are obtain for two different aspects of the volumetric expansions, Power law and an exponential expansion and discuss about its constrains. Here we also calculate the value of shear scalar (σ), Exponential scalar(θ), anisotropic parameter (Δ), Hubble Parameter (H), Deceleration Parameter (q). For exponential expansion it isobserved that energy density and pressure are positive for a specific constant and energy density is decreasing function of time. So it is behave like steady model of the universe at large time. Whereas for power law expansion, energy density and pressure are also positive for a specific constant. Where the energy density is decreasing function of time, and as time increase it tends to zero. As the value of $\frac{dP}{d\rho}$is positive, the model is stable.

Keywords: Consistency, Perfect Fluid, Plane Symmetry space time, f(R,T) Gravity, Stability.

INTRODUCTION

Regarding high red shift from the type Ia supernova and cosmic microwave background anisotropy, it is observe that the universe is accelerating [1-4]. In expansion of the universe not only late time accelerated expansion but also existence of dark energy (DE) and dark matter (DM) have received attention. From last some decays, the general theory of Relativity has been modified in many ways. Among the various modified theories f(R) gravity is most suitable to explain the exact nature of accelerated expansion of the universe. Its result are unification of early time inflation and late time acceleration.

Bertolami et al. [5] proposed new modified gravitational theories by coupling the arbitrary function of Ricci Scalar (R) with lagrangian matter density L_m. TiberiuHarko, Francisco S.N. Lobo, Shin'ichiNojiri, Sergei D. Odintsov[6] in 2011 extended this theory by coupling matter and geometry and also derived the field equation of $f(R,T)$gravity by section Swith respect to components of metric tensors g_{ij}.The modification of $f(R)$gravity is $f(R,T)$theory, where T is included as exotic non ideal matter configuration [6]. In this paper we work on f(R,T)gravity, where$f(R,T)$is given as

$$s = \int \sqrt{-g}\left(\frac{1}{16\pi G} f(R,T) + L_m\right) d^4x(1)$$

Where, $f(R,T)$is the arbitrary function of Ricci Scalar 'R'and trace 'T'of stress energy T_{ij} of the matter. $L_m$lagrangian matter density.

In the $f(R,T)$ gravity with respect to metric the variation of matter energy tensor can be considered. ThereforeAzizi [7] studied the woemhole solution. Naidu et at. [8] explore the Frieddman-Robertson-Walker(FRW) in $f(R,T)$ gravity. Whereas Reddy, Kumar[9] work on LRS Bianchi type II space time with perfect fluid in $f(R,T)$framework.Pawar et al. [10] studied an anisotropic stress tensor with spherically symmetrical fluid cosmological model in general relativity. The energy conditions are investigated by Sharif [11] with perfect fluid in $f(R,T)$ framework for FRW space time. Whereas it is demonstrated by Jamil et al. [12] for dust fluid. Houndjo [13] investigated models by using $f(R,T) = f_1(R) + f_2(T)$. Katore, Hatkar and Baxi [14] studied on hypersurface homogeneous model in $f(R,T)$gravity and check its stability. M.F. Shamir[15]investigate solution of Plane symmetric space time in $f(R,T)$ framework.V.R. Chirde, S.H. Shekh [16]studied dark energy models in the form of wet dark fluid with plane symmetry in $f(R,T)$gravity.

Motivated by above mention studies, we study the Plane symmetric space time with perfect fluid in $f(R,T)$gravity. For this we considered,

$$f(R,T) = f_1(R) + f_2(T)$$

Where,

$$f_1(R) = 1$$
$$f_2(T) = \mu T$$

With $T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\partial\sqrt{-g}}{\partial g^{ij}}L_m$

Here we obtain the field equation of the f(R,T)gravity model as

$$f_R(R,T)R_{ij} - \frac{1}{2}f_R(R,T)g_{ij} + (g_{ij}\nabla^\alpha\nabla_\alpha - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_R(R,T)\theta_{ij}$$

where $\theta_{ij} = -2T_{ij} - pg_{ij}, f_R = \frac{\partial f(R,T)}{\partial R}$, $f_T = \frac{\partial f(R,T)}{\partial T}$, ∇_i is the derivative and T_{ij} the standard matter energy momentum tensor derived from the Lagrangian L_m.

Apply this on plane symmetric space time matter as

$$ds^2 = -dt^2 + Adx^2 + B(dy^2 + dz^2)$$

Where the metric potential A and B are the function of 'x' and 't' only.

The energy momentum tensor for perfect fluid distributed is given by

$$T_j^i = (p + \rho)u_i u^j - p\delta_j^i$$

Where,$u^i$is flow vector satisfy $g_{ij}u^i u^j = 1$

ρ is total energy density of fluid

P is corresponding pressure.

The equation of state of perfect fluid is $p = \gamma\rho$ with $\gamma\epsilon[0,1]$

In next section, we obtain value of A and B , then study two volumetric expansion of the universe. Then observe the consistency by calculating its distances, parameters.

The f(R,T) gravity and field equation

The stress energy tensor of matter is as follows:

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\partial\sqrt{-g}}{\partial g^{ij}}L_m (2)$$

we assume the function f(R,T) as

$$f(R,T) = R + 2f_1(T) (4)$$

Where ,$f_1(T)$ is arbitrary function of Trace T.

Using the particular form of average scale factor Chuabey and Shukla [17] discussed the Bianchi type Ispace time in $f(R,T)$gravity, obtained a class of cosmological model. Ram et al. [18] gives a new class of exact solutions in presence of perfect fluid for special kind of function $f(R,T) = R + 2\lambda T$, where λ is a constant.

Consider plane symmetric space time metric as,

$$ds^2 = -dt^2 + Adx^2 + B(dy^2 + dz^2) (5)$$

Where the metric potential A and B are the function of 'x' and 't' only.

In General Relativity Pradhan & Pandya [19],[20], Pradhan et al. [19],[21], Pradhan & Kumar [22] and Pradhan & Ram [23] investigated Plane symmetric models for various kind of uses.

The energy momentum tensor for perfect fluid distributed is given by

$$T_j^i = (p + \rho)u_i u^j - p\delta_j^i (6)$$

Where,$u^i$is four velocity vector of fluid, with components(0,0,0,1) satisfying $u_i u^j = -1$.

ρ is total energy density of fluid and P is corresponding pressure. The equation of state of a perfect fluid is $p = \gamma\rho$ with $\gamma\epsilon[0,1]$. $0 \le \gamma \le 1$is necessary for the existence of local mechanical stability. Here the matter Lagrangian can be taken as $L_m = -p$.

We choose $f_1(T) = \mu T$.(7)

The corresponding field equation for the metric (7) can be written as follows,

$$\frac{{B_4}^2}{B^2} + \frac{2B_{44}}{B} = (8\pi + 2\mu)p + (5\mu p - \mu\rho) (8)$$

$$\frac{A_4 B_4}{AB} + \frac{B_{44}}{B} + \frac{A_{44}}{A} = (8\pi + 2\mu)p + (5\mu p - \mu\rho) (9)$$

$$\frac{{B_4}^2}{B^2} + \frac{2A_4 B_4}{AB} = -(8\pi + 2\mu)\rho + (5\mu p - \mu\rho) (10)$$

Where the subscript 4 is used to denote differentiation with respect to time t.

The volume of the universe is given as,

$$V = AB^2 (11)$$

From equation (8) and (9), we get ;

$$\frac{A}{B} = k_2 \exp(k_1 \int \frac{1}{V} dt) \ (12)$$

k_1, k_2 are constants.
From equation (12) we get,

$$A = D_2 V^{\frac{1}{3}} e^{X_2 \int \frac{1}{V} dt} (13)$$

And

$$B = D_1 V^{\frac{1}{3}} e^{X_1 \int \frac{1}{V} dt} (14)$$

As $X_2 + 2X_1 = 0 \ and \ D_1{}^2 D_2 = 1$
The directional Hubble parameter in the direction of the x,y and z axes are $H_x, H_y \ and \ H_z$ respectively.
Which are define as
$H_x = \frac{A_4}{A}, H_y = H_z = \frac{B_4}{B}$ (15)
The mean Hubble parameter is defined as follows,

$$H = \frac{1}{3}\frac{V_4}{V} = \frac{1}{3}\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) (16)$$

Anisotropic parameter is defined as follows;

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2}{9H^2}\left(\frac{A_4}{A} - \frac{B_4}{B}\right)^2 \ (17)$$

Where H_i (i=1,2,3) are the directional Hubble parameters in the direction of the x,y and z axes respectively. The expansion scalar $(\theta)$and shear scalar (σ) are defined as follows;

$$\theta = 3H = \left(\frac{A_4}{A} + \frac{2B_4}{B}\right) (18)$$

$$\sigma^2 = \frac{3}{2}\Delta H^2 = \frac{1}{3}\left(\frac{A_4}{A} - \frac{B_4}{B}\right)^2 \ (19)$$

As we have three independent equations (8)-(10) in unknowns p, ρ, A, B.
For complete solution of the system, we consider two volumetric expansions as, exponential and power, since the law of variation for the Hubble parameter proposed by Berman yields a constant value of the deceleration parameter. But for our observation this law is not consistent, so by choosing new variation of Hubble parameter is proposed, which led to two volumetric expansions,which are defined as follows;

$$V = C_2 e^{3mt} (20)$$

$$V = C_3 t^{3m} (21)$$

Where $C_2, C_3, and \ m$ are arbitrary positive constants. When $0 < m < 1$,, the power law model consist deceleration parameter q, whereas when $m > 1$ shows accelerated expansion . For q = 0 and m = 1 it shows inflationary universe for q=0 and m=1. The accelerating volumetric expansion is given by exponential model.

EXPONENTIAL EXPANSION

For volumetric exponential expansion we obtain;

$$A = D_2 C_2^{\frac{1}{3}} exp\left(mt - \frac{2K_1}{9C_2 m} e^{-3mt}\right) \qquad (22)$$

$$B = D_1 C_2^{\frac{1}{3}} exp\left(mt + \frac{K_1}{9C_2 m} e^{-3mt}\right) (23)$$

It is observed that, in the early stage of the universe, scale factor are constant and increase very slowly for positive value of m, and suddenly expanded to a large extent,which is consistent to Big Bang scenario. Similar results was obtained by Singh and Beesham [24] and byKatore and Hatkar[25] .

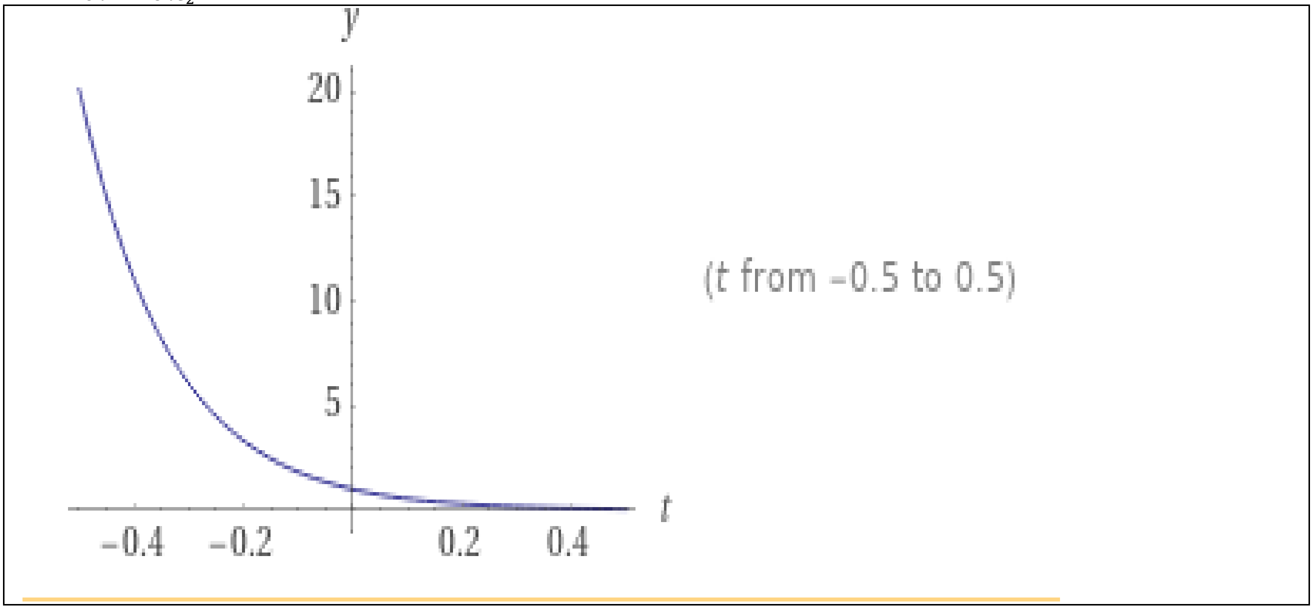
For this model energy density and pressure which are obtained are as follows,
$\rho = Not \ defined$ for negative value of μ

Therefore by considering its absolute value we get

$$\rho = \frac{15m^2}{16\pi} + \frac{5{K_1}^2}{48\pi {C_2}^2} e^{-6mt} (24)$$

and

$P = \frac{3m^2}{16\pi} + \frac{{K_1}^2}{48\pi {C_2}^2} e^{-6mt} (25)$



Plot of Energy density versus Cosmic time for $C_2 = m = K_1 = 1$

For above equations, we observed that energy density and pressure are positive for a specific constant. Where the energy density function is decreasing function of time. Hence it is behave as steady model of the universe at large time. This steady ness of the model is investigated by using the sign of ratio $\frac{dP}{d\rho}$. Here the value of $\frac{dP}{d\rho} = \frac{1}{5} > 0$ and hence the model is stable.

The values of expansion scalar θ,shear saclar σ, and deceleration parameter q , anisotropic parameter Δ are obtained as follows:

$$\theta = 9m (26)$$

$$\sigma = \frac{K_1 e^{-3mt}}{\sqrt{3} C_2} (27)$$

$$q = -1 (28)$$

$$\Delta = \frac{2}{81} \frac{{K_1}^2}{m^2 {C_2}^2} e^{-6mt} (29)$$

The value q is negative, this indicates that the universe is accelerated. The rate of expansion of universe is constant as the value of expansion scalar is constant. At early stage ot the evolution the ratio of shear scalar and expansion scalar was nonzero, it tends to zero as time increase. Hence Initiallythe universe was anisotropic and at late time it approaches isotropy.

Power law model

For volumetric exponential expansion we obtain;

$$A = D_2 C_3^{\frac{1}{3}} t^m exp\left(\frac{2K_1}{3C_3(1-3m)} t^{1-3m}\right) (30)$$

$$B = D_1 C_3^{\frac{1}{3}} t^m exp\left(\frac{-K_1}{3C_3(1-3m)} t^{1-3m}\right) \quad (31)$$

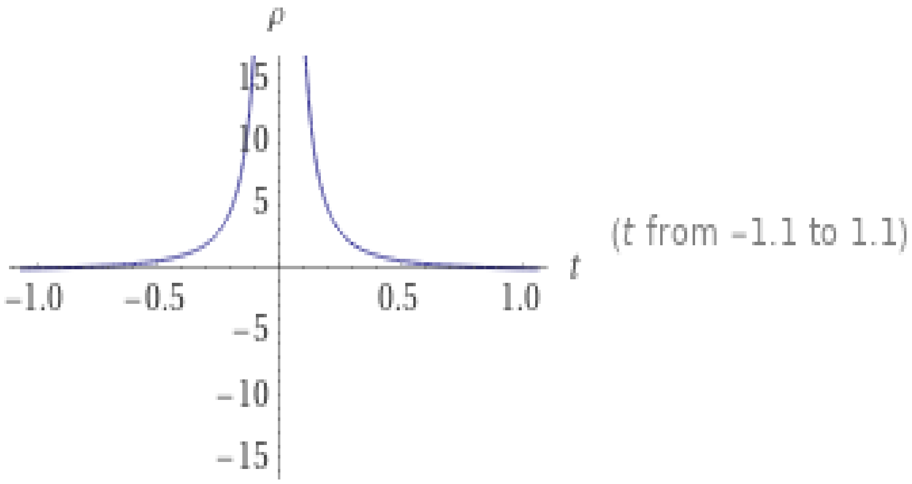
It is observed that, the model has initial singularity at t=0, as the value of A and B vanishes at that point.After passing some time its value increases which satisfy the complete agreement with the universe of Bing Bang.The result is similar to Katore [14]. Moreover the solution of field equation is obtained for value $\gamma = \frac{3}{7}, m = \frac{1}{3} with\ \mu = 4\pi$.

For this model energy density and pressure which are obtained are as follows,

$$\rho = \frac{7}{114\pi}\left(-1 + \frac{{K_1}^2}{{C_3}^2}t^{-2}\right) (32)$$

and

$$P = \frac{49}{342\pi}\left(-1 + \frac{{K_1}^2}{{C_3}^2}t^{-2}\right) (33)$$



Plot of Energy density versus Cosmic time for $C_3 = K_1 = 1$

For above equations, we observed that energy density and pressure are positive for a specific constant. Where the energy density function is decreasing function of time, and as time increase it tends to zero.

This steady ness of the model is investigated by using the sign of ratio $\frac{dP}{d\rho}$. Here the value of $\frac{dP}{d\rho} > 0$ and hence the model is stable.

The values of expansion scalar θ,shear saclar σ, and deceleration parameter q , anisotropic parameter Δ are obtained as follows:

$$\theta = \frac{9m}{t} (34)$$

$$\sigma = \frac{K_1 t^{-3m}}{\sqrt{3}C_3} (35)$$

$q = -\frac{2}{3}$; for m=1

And q = -1; for m=0 (36)

$$\Delta = \frac{2}{81}\frac{{K_1}^2}{m^2 {C_3}^2}t^{-6m+2} (37)$$

The value q is negative, this indicates that the universe is accelerated. The rate of expansion of universe is constant as the value of expansion scalar is constant.

Physical Consistency of exponential model

We investigate the physical consistency of the model with the observational constrains. By measuring the parameters such as look back time, proper distance, luminosity distance, angular diameter, etc.

A. The Look Back Time

It is denoted by t_L and defined as the elapsed time between time of the universe t, when the light of cosmic source at a specific red shift z was emitted and its present time t_0 .

In our exponential model context, we observe that;

$$t_L = t_0 - t = \int_a^{a_0} \frac{da}{a}$$

Where a_0 is the scale factor of the universe for present day and its value is $\frac{a_0}{a} = 1 + z$
For model of exponential law

$$a = C_2^{\frac{1}{3}} e^{mt}$$

$$z + 1 = \frac{a_0}{a} = \frac{C_2^{\frac{1}{3}} e^{mt_0}}{C_2^{\frac{1}{3}} e^{mt}} = e^{m(t_0 - t)}$$

$$\ln(z+1) = m(t_0 - t)$$

$$t_0 - t = \frac{1}{m}\ln(z+1)$$

$$H_0(t_0 - t) = \frac{H_0}{m}\ln(z+1)$$

$t = t_0 - \frac{\ln(z+1)}{H_0}$, $\quad H_0$ *is present Hubble parameter*

B. Proper Distance

$d(z) = {H_0}^{-1} z$

It is defined as the distance between cosmic source emtting light at any instant $t = t_0$ located at $r = r_1$ with red shift z and it observe at r = 0 and $t = t_0$receving the light from the source emitted i.e.,

$$d(z) = r_1 a_0$$

Where $r_1 = \int_t^{t_0} \frac{dt}{a} = {H_0}^{-1} {a_0}^{-1} z$

T

he present distance is linear with red shift z. Also at z = ∞

It is always infinite

C. uminosity Distance

It is denoted d_L and defined as,

$$d_L = \left(\frac{L}{4\pi l_*}\right)^{\frac{1}{2}} r_1 a_0 (1+z)$$

Where L is the absolute value and l_* is the apparent luminosity source.

$$d_L = d(z)(1+z)$$

Hence,

$$d_L = {H_0}^{-1} z(1+z)$$

D. Angular Diameter

The source of light of diameter D at $r = r_1$ and $t = t_1$

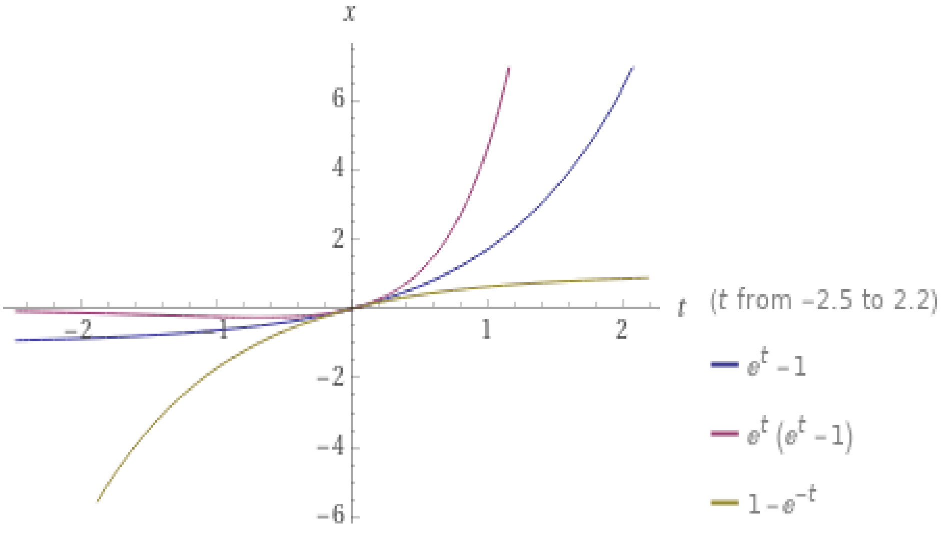
Observed at $r = 0$and $t = t_0$, which is given as,

$$\delta = \frac{D}{r_1 a(t_1)} = \frac{D(1+z)^2}{d_L}$$

$d_L$is the ratio of the source of diameter to its angular diameter,

$$d_A = \frac{D}{\delta} = \frac{d_L}{(1+z)^2}$$

$$d_A = \frac{{H_0}^{-1} z}{(1+z)}$$



Proper distance, Luminosity distance, Angular distance versus time t for Exponential model

The Proper distance, Luminosity distance and Angular distance increase as increase in time

Physical Consistency of Power law Model

To investigate the consistency of the model of power law, we measure the physical parameter such as red shift, look back time, proper distance, luminosity distance, angular distance etc.

A. Look Back time

We have, $R = C_3^{\frac{1}{3}} t^m$

$$(z+1) = \frac{a_0}{a} = \frac{C_3^{\frac{1}{3}} {t_0}^m}{C_3^{\frac{1}{3}} t^m}$$

$$\frac{t^m}{{t_0}^m} = \frac{1}{(z+1)}$$

$$t^m = {t_0}^m (z+1)^{-1}$$

The age of the universe is

$$m{H_0}^{-1} = t_0$$

e.i.,$H_0(t_0 - t) = m(1-(1+z)^{\frac{-1}{m}})$

for large value of z;

$$H_0(t_0 - t) \approx m$$

And for small value of z;

$$H_0(t_0 - t) \approx 0$$

B. Proper distance

$$d(z) = a_1 {H_0}^{-1}(1-(1+z)^{1-\frac{1}{m}})$$

C. Luminosity distance

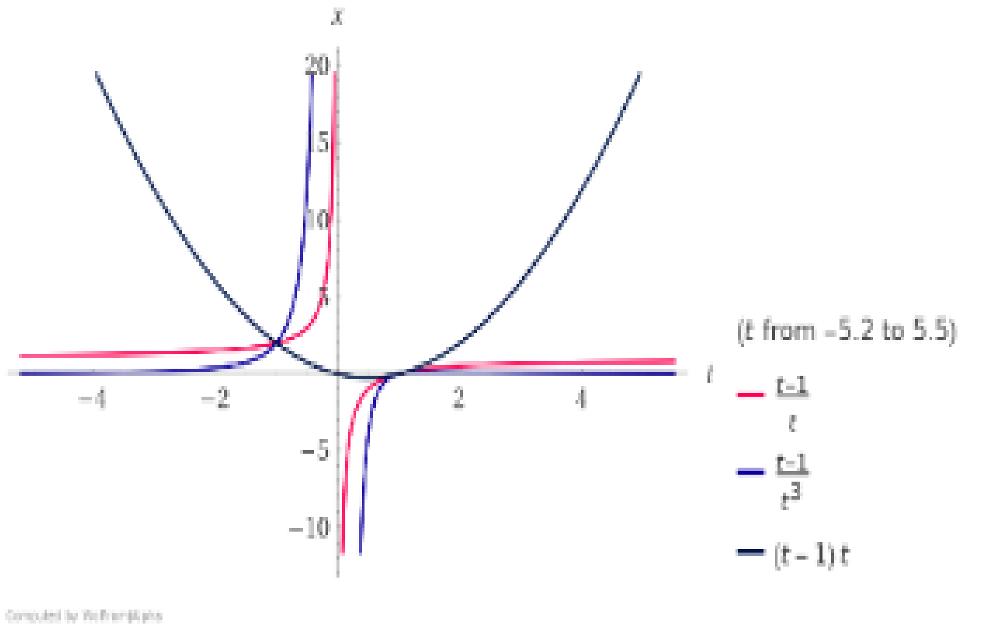
$$d_L = a_1 {H_0}^{-1}\left[1-(1+z)^{1-\frac{1}{m}}\right](1+z)$$

D. Angular Diameter

$$d_A = d_L(1+z)^{-2}$$

$$d_A = a_1 {H_0}^{-1}\left[1-(1+z)^{1-\frac{1}{m}}\right](1+z)^{-1}$$

The luminosity distance is linear with respect to red shift, while proper distance some constant value with the increment in the red shift.



Proper distance, Luminosity distance, Angular distance versus time t for Power law model

CONCLUSION

In present work, we find in the f(R,T) gravity , the prefect fluid cosmological model for plane symmetry space time. This kind of work already done by Shamir [15] for desitter and antidesitter space time in general relativity.

Here in this paper, here we observed that, the universe is accelerating and very closed to isotropy at large time. For exponential expansion our calculated values of density and pressure are same as in hypersuface homogeneous cosmology by Katore [14].

The value of pressure and density are positive for specific constant, hence at large time the model is just like a steady state model of the universe. In power law solution, as scale factors diverge to infinity at the large time there will be Big Rip at least as far in the future. For m>1, the deceleration parameter is negative. Hence the model is consistent with the cosmological observations.

Furthermore in the exponential expansion, at initial time constant value is attain by scale factor. This is consist with Big Bank scenario which is resembles to results of Katoreand shaikh [26].

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